Regularity of Lyapunov Exponents for Random Products of Matrices: Recent Advances

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Let μ be a compactly supported probability measure on $GL(d,\mathbb{R})$, and let $(A_n)_{n\geq 1}$ be a sequence of independent, identically distributed random matrices with common law μ. We consider the random products

$$L_n = A_n \cdots A_1.$$

By the multiplicative ergodic theorem, the limits

$$\lambda_i(\mu) = \lim_{n\to\infty} \frac{1}{n} \log \sigma_i(L_n),$$

where $\sigma_i(L_n)$ denote the singular values of L_n, exist almost surely and define the Lyapunov spectrum

$$\lambda_1 \geq \cdots \geq \lambda_d.$$

A central problem is to understand how the Lyapunov exponents depend on the probability measure μ. In the strongly irreducible and proximal setting, classical results establish continuity and Hölder regularity of the top Lyapunov exponent. These results rely on Furstenberg's theory, spectral gap properties of associated transfer operators, and quantitative large deviation estimates.

In this talk, we first survey foundational results in the irreducible framework. We then turn to the reducible case, where invariant subspaces may appear. In this setting, the behavior of Lyapunov exponents can be significantly more delicate: regularity may fail at parameters where reducibility changes, spectral gaps close, or exponents collide. We discuss recent advances that clarify how regularity properties extend in the presence of invariant splittings.

The aim of the talk is to present a coherent picture of regularity phenomena for Lyapunov exponents in the general d-dimensional setting.