Counting elliptic curves over $\mathbb{Q}$ with bounded naive height

Daniel Mora[*], Adrián Barquero[†]

Abstract

In this lecture we will give exact and asymptotic formulas for counting elliptic curves $E_{A,B}\colon y^2 = x^3 + Ax + B$ with $A, B \in \mathbb{Z}$, ordered by naive height. We study the family of all such curves and also several natural subfamilies, including those with fixed j-invariant and those with complex multiplication (CM). In particular, we will show formulas for two commonly used normalizations of the naive height appearing in the literature: the *calibrated naive height*, defined by

$$H^{\mathrm{cal}}(E_{A,B}) := \max\{4|A|^3, 27B^2\},$$

and the *uncalibrated naive height*, defined by

$$H^{\mathrm{ncal}}(E_{A,B}) := \max\{|A|^3, B^2\}.$$

As part of our approach, we give a completely explicit parametrization of the set of curves $E_{A,B}$ with fixed j-invariant and bounded naive height, describing them as twists of the curve E_{A_j,B_j} of minimal naive height for the given j-invariant. This work is the result of a joint work with professor Adrián Barquero.

[*]CIMPA, Universidad de Costa Rica. e-mail: `daniel.moramora@ucr.ac.cr`
[†]CIMPA, Universidad de Costa Rica. e-mail: `adrian.barquero_s@ucr.ac.cr`