Iwasawa invariants and uniform distribution

Abstract

Let $K/\mathbb{Q}$ be a finite extension and p a fixed prime number. There exists an extension K_∞/K, called the cyclotomic $\mathbb{Z}_p$-extension, such that its Galois group $\mathrm{Gal}(K_\infty/K)$ is isomorphic to the group of p-adic integers $\mathbb{Z}_p$. For every n there exists a unique intermediate field K_n such that $\mathrm{Gal}(K_n/K) \cong \mathbb{Z}/p^n\mathbb{Z}$. If we let A_n denote the p-part of the class group of K_n and $|A_n| = p^{e_n}$, it is a well-known result due to Iwasawa that there are invariants $\lambda, \mu \geq 0$ and $\nu \in \mathbb{Z}$ such that

$$e_n = \lambda n + \mu p^n + \nu.$$

The goal of this talk is to discuss a well-known result proved by Washington that uses uniform distributions to show that $\mu = 0$ (the growth is linear) whenever $K/\mathbb{Q}$ is abelian. There are similar conjectures for the λ invariant, and for invariants over elliptic curves. We will analyze how you can use similar ideas in these different situations.