

# ITÔ'S FORMULA FOR ITÔ PROCESSES DEFINED WITH RESPECT TO A CYLINDRICAL MARTINGALE-VALUED MEASURE

D. CAMPOS <sup>2</sup>

In this talk, we present an Itô formula for Hilbert space-valued Itô processes driven by cylindrical martingale-valued measures, based on our recently developed stochastic integration framework (see [7]). We outline the main ideas behind the construction and highlight the analytical tools that make the results possible, including the predictable and optional quadratic variation of stochastic integrals, the decomposition into continuous and purely discontinuous components, and a Riemann representation formula. We then discuss an application of this formula, namely a Burkholder-type inequality for stochastic integrals with respect to cylindrical martingale-valued measures.

The talk is based on a join work with S. Cambronerio <sup>1</sup>, C. A. Fonseca-Mora <sup>3</sup> and D. Mena <sup>4</sup>

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CENTRO DE INVESTIGACIÓN EN MATEMÁTICA PURA Y APLICADA, ESCUELA DE MATEMÁTICA, UNIVERSIDAD DE COSTA RICA

*E-mail address:* <sup>1</sup> [santiago.cambronerio@ucr.ac.cr](mailto:santiago.cambronerio@ucr.ac.cr), <sup>2</sup> [josedavid.campos@ucr.ac.cr](mailto:josedavid.campos@ucr.ac.cr),

<sup>3</sup> [christianandres.fonseca@ucr.ac.cr](mailto:christianandres.fonseca@ucr.ac.cr), <sup>4</sup> [dario.menaarias@ucr.ac.cr](mailto:dario.menaarias@ucr.ac.cr).

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